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Heterogeneity of Expectations and Financial Crises

—A Stochastic Dynamic Approach—

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Abstract

This study presents a stochastic dynamic model which builds theoretically on the concept of "the liquidity trap" *à la* Keynes. The model demonstrates that money holding becomes relatively advantageous as the proportion of money holders increases and that such a situation may bring about multiple equilibria—an "excess liquidity equilibrium" and a "liquidity crisis equilibrium." Through comparative statics, the study finds that the heterogeneity of expectations plays a crucial role for the existence of multiple equilibria. The model helps to explain the recent financial crisis and offers meaningful implications for monetary policies, particularly concerning the effectiveness of unconventional monetary policies.

JEL Classification: E12, E41, E44, E52

Keywords: heterogeneity of expectations, liquidity trap, Keynes, financial crisis, unconventional monetary policies

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1 Introduction

The subprime financial crisis has had a devastating impact on the world economy since 2007. Its causes and authorities' policy responses remain widely debated among scholars, politicians, and business people.

This study presents a stochastic dynamic model which views the crisis as a resurgence of "the liquidity trap" postulated by Keynes (1936). A liquidity trap represents a situation in which the demand for money is ultra sensitive to the interest rate when it is commonly regarded as too low to decline farther. It is noteworthy that the occurrence of a liquidity trap is contingent on people's expectations about the future interest rate. Particularly, the liquidity trap emerges when a plurality of economic agents uniformly predict that the interest rate will not drop to a lower level. Keynes (1936, p.172) remarks, "It is interesting that the stability of the system and its sensitiveness to changes in the quantity of money should be so dependent on the existence of a *variety* of opinion about what is uncertain." In other words, a liquidity trap is a situation in which the heterogeneity of opinions is absent in the economy.

Following the lead of Keynes, Tobin (1958) developed models of the demand for money that take two approaches. One is his famous mean-variance approach that laid the foundation for the theory of portfolio selection and has been fruitful for subsequent studies in financial economics. His other approach focuses on the heterogeneity of expectations about the interest rate. When each investor has a different expectation about the future interest rate, the demand for money can be traced on a smooth, downward sloping curve, the shape of which depends on the distribution of expected interest rates. However, the second approach, in particular, the heterogeneity of expectations, has received little attention in subsequent developments of macroeconomics and financial economics. Although several studies have pointed out the importance of the heterogeneity in opinions, few economic models sufficiently incorporate it. In particular, the majority of mainstream macroeconomic models presuppose the representative agent as well as the rational expectations hypothesis.

This study deploys a binary choice model, following Aoki (1998) and Aoki and Yoshikawa (2006). In this model, an economic agent stochastically changes her decisions. The transition rates from one state to the other vary, depending on the degree of diversity in expectations. Applying this model to the money/bond choice, this study seeks to derive the money demand function discussed by Keynes and analyze how the heterogeneity of expectations affects it.

The study also examines the case in which an increase in the ratio of money holders makes money holding more profitable. Many preceding studies indicate the possibility of such cases. However, this study distinguishes itself from those by showing that in such a case, the economy may have multiple equilibria, stochastically settle on one, and transit from one to the other over time. Defining one equilibrium as an "excess liquidity equilibrium" and the other as a "liquidity crisis equilibrium," this study interprets financial crises as a leap from the former to the latter. Further, comparative statics of the model clarifies the condition for the existence of multiple equilibria.

The model helps to explain the recent financial crisis and provides meaningful implications for monetary policy. In particular, in analyzing the influences of heterogeneous expectations on the economy, the model offers a novel perspective about the monetary policy. As a result, the study reaches the proposition that conventional monetary policies are not effective, but rather unconventional monetary policies are.

This paper proceeds as follows. Section 2 constructs a stochastic dynamic model, and Section 3 derives the money demand function from the model. Section 4 analyzes the case in which an increase in the proportion of money holders amplifies the profitability of money holding. Section 5 discusses implications of our findings for financial crises and monetary policy. Section 6 concludes the paper by suggesting possible extensions of the study.

2 The Model

2.1 Binary Choice Model

Suppose an economy with N fixed number of agents, each of whom has two choices at any time and each of whom is classified according to her choice. Let n be the number of agents who have chosen Decision 1. Then, the remaining $N - n$ agents have chosen

Decision 2. We define $x \equiv \frac{n}{N}$ for notational simplicity.

Let us denote the transition rate from Decision 2 to Decision 1 as $w_{n,n+1}$, and that from Decision 1 to Decision 2 as $w_{n,n-1}$. Further, $w_{n,n+1}$ and $w_{n,n-1}$ are specified as[1]

$$w_{n,n+1} = \lambda(1-x)\eta_1(x), \tag{1}$$

$$w_{n,n-1} = (1-\lambda)x\eta_2(x). \tag{2}$$

λ $(0 < \lambda < 1)$, the birth rate in the standard birth-and-death process, signifies the superiority of Decision 1 regardless of the value of x. On the other hand, $\eta_1(x)$ and $\eta_2(x)$ represent the superiority of Decision 1 and Decision 2 that are affected by the value of x. If η's are one, agents independently decide to switch their choices.

We denote the probability of n agents choosing Decision 1 at time t as $P_t(n)$. Keeping track of inflows and outflows of the probability flux, we obtain

$$P_{t+\Delta t}(n) = P_t(n-1)w_{n-1,n}\Delta t + P_t(n+1)w_{n+1,n}\Delta t - P_t(n)(w_{n,n+1} + w_{n,n-1})\Delta t + o(\Delta t). \tag{3}$$

When Δt converges to 0, equation (3) gives rise to the following master equation:

$$\frac{dP_t(n)}{dt} = P_t(n-1)w_{n-1,n} + P_t(n+1)w_{n+1,n} - P_t(n)(w_{n,n+1} + w_{n,n-1}) \tag{4}$$

for $n = 1, 2, \ldots, N-1$.[2] Let $\pi(n)$ be the limit of $P_t(n)$ when $t \to \infty$. Then equation (4) reduces to the following detailed balance condition,

$$\pi(n)w_{n,n-1} = \pi(n-1)w_{n-1,n}, \quad 1 \leq n \leq N. \tag{5}$$

Then, transforming (5), we obtain

$$\frac{\pi(n)}{\pi(n-1)} = \frac{w_{n-1,n}}{w_{n,n-1}} = \frac{\lambda(N-n+1)}{(1-\lambda)n} \cdot \frac{\eta_1(\frac{n-1}{N})}{\eta_2(\frac{n}{N})},$$

[1] Transition rates should be specified as $w_{n,n+1} = \lambda(N-n)\eta_1(x) = \lambda N(1-x)\eta_1(x)$, $w_{n,n-1} = (1-\lambda)n\eta_2(x) = (1-\lambda)Nx\eta_2(x)$. However, if the unit of time is properly manipulated, specifications as (1) and (2) are also valid.

[2] We need the boundary condition for $n = 0$ and $n = N$,

$$\frac{dP_t(0)}{dt} = P_t(1)w_{1,0} - P_t(0)w_{0,1}, \quad \frac{dP_t(N)}{dt} = P_t(N-1)w_{N-1,N} - P_t(N)w_{N,N-1}.$$

which yields

$$\pi(n) = \Big(\frac{\lambda}{1-\lambda}\Big)^n \pi(0) \binom{N}{n} \prod_{i=1}^{n} \frac{\eta_1(\frac{i-1}{N})}{\eta_2(\frac{i}{N})}.$$

Taking the natural logarithm of both sides,

$$\ln \pi(n) = n \ln \frac{\lambda}{1-\lambda} + \ln \pi(0) + \ln \binom{N}{n} + \sum_{i=1}^{n} \ln \frac{\eta_1(\frac{i-1}{N})}{\eta_2(\frac{i}{N})},$$

we approximate this equation when N is sufficiently large by[3]

$$\ln\ \pi(x) \approx NH(x) + Nx \ln \frac{\lambda}{1-\lambda} + \ln \pi(0) + \int_0^x \ln \frac{\eta_1(y)}{\eta_2(y)} dy,$$

where $H(x) \equiv -x \ln x - (1-x) \ln(1-x)$. Maximizing $\pi(x)$, we obtain

$$-\ln \frac{\eta_1(x)}{\eta_2(x)} - N \ln \frac{\lambda}{1-\lambda} = N \ln \frac{1-x}{x}. \tag{6}$$

The value of x satisfying (6) is the value that is the most probable in the stationary distribution.

Equation (6) offers an intuitive interpretation as follows. The left-hand side is 0 if each transition rate is equal. The greater the transition rate from Decision 2 to Decision 1 (i.e., the greater the superiority of Decision 1) is, the smaller the left-hand side is. That is, the left-hand side indicates the strength of the circumstance which leads economic agents to choose the superior option. This corresponds to the incentive mechanism in conventional economics.

Meanwhile, the right-hand side is 0 if $x = \frac{1}{2}$, monotonically decreasing in x. In a sense, the right-hand side expresses the mechanism through which x approaches $\frac{1}{2}$. It should be called the "entropy effect" because it is the effect which forces a stochastic system close to the most probable case.[4]

[3]When N is large, we can use Stirling formula,

$$N! \approx \sqrt{2\pi N}\Big(\frac{N}{e}\Big)^N,$$

and then we obtain

$$\ln \binom{N}{n} = NH(\frac{n}{N}) + O(\frac{1}{N}).$$

[4]Existence of the entropy effect is exemplified by air. If particles of air were influenced only by

2.2 Choice between money and bond

Suppose that economic agents hold assets in the form of money or bonds. Let V_m be the return rate of money, and V_b the return rate of bonds. These values are assumed to be common knowledge. Let U_m and U_b be the utilities of holding money and bonds, respectively. They are defined as

$$U_m = V_m + \varepsilon_m, \quad U_b = V_b + \varepsilon_b. \tag{7}$$

Here, ε_m and ε_b are random variables.[5]

The above formation, which decomposes utility into two parts, is justified by the following reasoning. First, an economic agent may have private information about the future return rate. Morris and Shin (2002) adopt this formulation to analyze the situation in which economic agents determine their actions after observing both public and private signals. Second, an economic agent may hold money or bonds for purposes other than returns. Keynes (1937) points out that prices of capital-assets may move also on the basis of uncertainty, convention, and the other elements. If the expected utility given by factors other than rates of return is heterogeneous, utilities should be formulated as (7). In this case, the expected utilities of holding money or bonds have the variance caused by random variables, ε_m and ε_b.

An agent's choice hinges on the difference in utility given by holding money or bonds, $U_m - U_b$. From (7), we obtain,

$$U_m - U_b = -r - \varepsilon \tag{8}$$

where $r \equiv V_b - V_m$ represents the excess of the rate of return of bonds over money. For simplicity, we call it the nominal interest rate. We also set $\varepsilon \equiv \varepsilon_b - \varepsilon_m$. ε signifies an agent's personal assessment of the advantage of holding bonds instead of money. The assessment is based on her expectations regarding the future interest rate in addition to the common knowledge of the nominal interest rate.

gravitational force, air would not exist at any point above a certain height. However, air in fact exists at high altitudes although it is very thin. This is because not only gravity but also the entropy effect affects particles in air.

[5] Domencich and McFadden (1975) explain the binary choice model in which utilities are divided into a non-stochastic component and a stochastic component as in (7).

Hereafter, we assume that ε follows the logistic distribution.[6] That is,

$$\Pr(\varepsilon \leq z) = \frac{1}{1 + e^{-z/\gamma}}. \tag{9}$$

Equation (9) shows that the value of γ ($\gamma > 0$) produces its effect on the distribution of ε. When γ is small, the distribution of ε is dense in the neighborhood of 0. This implies that people's expectations tend to converge. Conversely, when γ is large, the distribution of ε, and hence people's expectations, tend to disperse. Thus, γ is a parameter which indicates the heterogeneity of agents' expectations. The smaller its value is, the smaller the variety of expectations is. From now on, we call γ *the heterogeneity of subjective expectations*, or simply *the heterogeneity of expectations*.

Interpreting x as the ratio of economic agents who hold money, we can analyze the movement of x by the model presented in Section 2.1. Specifically, some initial value of x provides the probability of judging that money is preferable to bonds, $\Pr(U_m - U_b \geq 0)$. Hence, we get the stationary distribution by setting

$$\eta_1 = \Pr(U_m - U_b \geq 0), \quad \eta_2 = \Pr(U_m - U_b < 0). \tag{10}$$

From (8),(9), and (10), we obtain

$$\eta_1 = \Pr(\varepsilon \leq -r) = \frac{e^{-r/2\gamma}}{e^{r/2\gamma} + e^{-r/2\gamma}}, \quad \eta_2 = \Pr(\varepsilon > -r) = \frac{e^{r/2\gamma}}{e^{r/2\gamma} + e^{-r/2\gamma}}. \tag{11}$$

Then, we can determine the most probable value of x in the stationary distribution.

3 Heterogeneity of expectations and the money demand function

3.1 Derivation of the money demand function

Solving equations (6),(10), and (11) simultaneously, we find that $\pi(x)$ is maximized when x satisfies

$$-\ln\frac{\eta_1}{\eta_2} = \frac{r}{\gamma} = N\Big(\ln\frac{1-x}{x} + \ln\frac{\lambda}{1-\lambda}\Big). \tag{12}$$

[6]The normal distribution can be assumed, but the analysis would be more complex because two parameters are needed. This assumption is not crucial for the results.

Fig.1 describes this situation. The intersection of the curved line with the horizontal line is the equilibrium value of x.

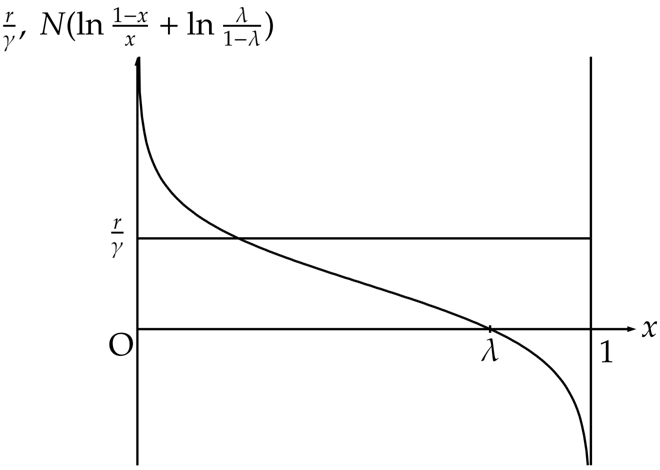

Fig.1

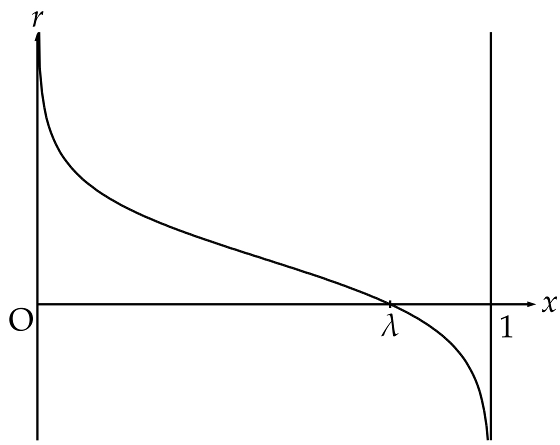

Fig.2

Solving (12) for x yields

$$x = \frac{\lambda}{\lambda + (1-\lambda)\exp(\frac{r}{\gamma N})}. \tag{13}$$

Fig.2 plots this function with r on the vertical axis and x on the horizontal axis. Since this figure displays the relation between the nominal interest rate and the proportion of individuals holding money, it is merely the money demand curve in a standard macroeconomics text.[7]

The intersection of the curved line with the horizontal axis is λ. Because λ indicates the superiority of money holding, the ratio of money holders, x, should be λ when the rates of return on bonds and money are both zero, and hence the nominal interest rate, r, is zero. Hereafter, we call λ *the liquidity preference effect*.

3.2 Heterogeneity of expectations and the money demand function

Tobin (1958) is among the most influential studies that elaborates on the derivation

[7]If we set the non-negativity constraint on the nominal interest rate, we need to find solutions in the range of $0 \leq x \leq \lambda$. But our model allows the nominal interest rate to be negative.

of the money demand function discussed by Keynes. He shows that when investors have diverse expectations regarding interest rates, the demand for money is a smooth, continuous curve the shape of which depends on the distribution of expected interest rates. Restated in the framework of our model, Tobin's study effectively argues that the money demand function is affected by the value of γ. Fig.3 depicts the shape of the money demand curves with different values of γ.

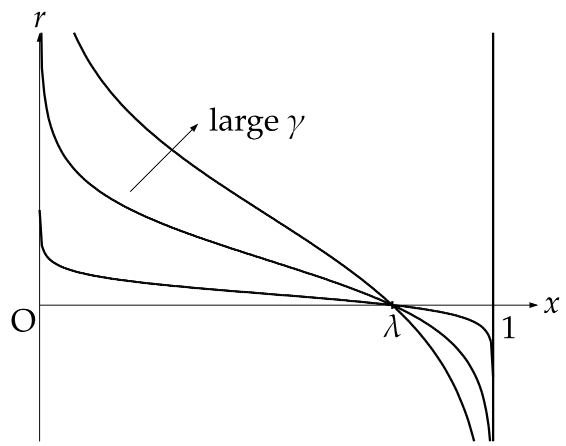

Fig.3

First, consider the situation in which γ is relatively small. The interest elasticity of money demand is small for large values of r, and vice versa. Particularly, when r is nearly zero, the money demand curve approaches horizontal. That is, the interest elasticity approaches infinity. As we explained in Section 2.2, agents' expectations are less various when the value of γ is small. Therefore, in such a situation, most market participants expect the interest rate to rise or the bond price to fall. This is the *liquidity trap* postulated by Keynes.

When γ is relatively small, more people want to hold bonds as far as returns on bond exceed those on money. However, as the difference in return decreases, people become indifferent between bonds and money. Due to the entropy effect, the ratio of money holders becomes equal to λ when people become totally indifferent between holding money and bonds.

Second, let us consider the case in which γ is relatively large. The money demand curve approaches vertical and the interest elasticity approaches zero. Because the heterogeneity in expectations is large, the demand for bonds does not increase substantially, nor does the demand for money decrease substantially as the interest rate rises. This suggests that the demand for money does not react sufficiently to the fluctuation in the interest rate. Hence, a huge movement in the interest rate is required for the market to be cleared.

4 Field effects and multiple equilibria

4.1 Field effects

This section introduces field effects into the model presented in the previous section. In this study, field effects are defined as the aggregate effect caused by a whole population or a fraction of microeconomic units.[8]

When field effects are introduced into the model, the value of x influences each agent's decisions. This study focuses on the case in which an agent mimics the action taken by the larger number of other agents. That is, we, in the preceding model, focus on the case where

$$\eta_1'(x) > 0, \; \eta_2'(x) < 0. \tag{14}$$

4.2 Modification of the model with field effects

The model in the previous section considers the case in which utilities of holding money or bonds are determined independently of the proportion of money holders. This section incorporates field effects into the model to examine the situation in which money holding becomes more attractive as the ratio of money holders increases.

Let us now assume that

$$U_m - U_b = -r + \ell(x) - \varepsilon, \tag{15}$$

[8]This definition is drawn from Aoki (1996, Ch.5). It can be interpreted as Marshallian externalities in microeconomics.

where the definitions of r and ε are the same as those in the previous section. $\ell(x)$ above represents the liquidity premium—the benefit from holding money. Now that field effects are incorporated, equation (13) holds here. Further, from $\eta_1(x) = \Pr(U_m - U_b \geq 0) = \Pr(\varepsilon \leq -r + \ell(x))$, we obtain

$$\ell'(x) > 0.$$

Section 5.2 will explain in detail the economic meaning of the above inequality.

Since (14) holds, (11) becomes

$$\Pr(U_m - U_b \geq 0) = \Pr(\varepsilon \leq -r + \ell(x)) = \frac{e^{(-r+\ell(x))/2\gamma}}{e^{(-r+\ell(x))/2\gamma} + e^{-(-r+\ell(x))/2\gamma}}.$$

Then, (12) can be written as

$$\frac{r - \ell(x)}{\gamma} = N\Big(\ln\frac{1-x}{x} + \ln\frac{\lambda}{1-\lambda}\Big). \tag{16}$$

For analytical simplicity, in what follows we specify $\ell(x)$ as

$$\ell(x) = \alpha x, \quad \alpha > 0.$$

Here, α is the parameter which indicates the degree of field effects on money holding. We call α simply *the field effect*. Hence, (15) becomes

$$\frac{r - \alpha x}{\gamma} = N\Big(\ln\frac{1-x}{x} + \ln\frac{\lambda}{1-\lambda}\Big). \tag{17}$$

4.3 Existence of multiple equilibria

Fig.4 is the graphical representation of (16). It schematically shows there are three equilibria. Fig.5 illustrates the relationship between the nominal interest rate, r, and the ratio of money holders, x. It can be considered as the money demand function analogous to Fig.2 in Section 3. However, Fig.5 exhibits three equilibria for one nominal interest rate within a certain range.

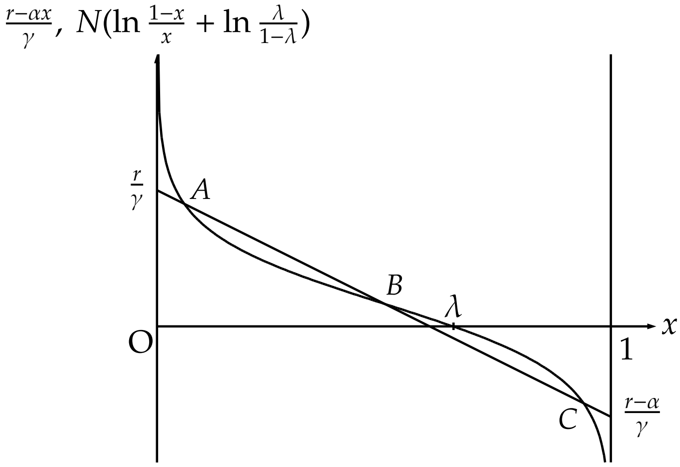

Fig.4

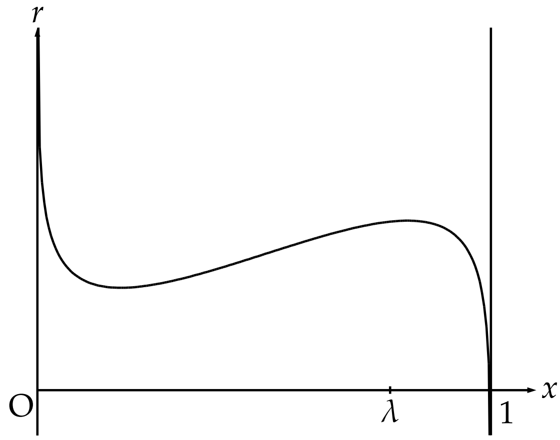

Fig.5

We now turn to the stability of equilibria. Equilibrium is stable if

$$-\frac{\ell'(x)}{\gamma} > \frac{d}{dx}\Big(N\ln\Big(\frac{1-x}{x}+\ln\frac{\lambda}{1-\lambda}\Big)\Big).$$

The stability is assured when $\ell'(x) = 0$ as shown in Section 3. When $\ell'(x) = \alpha > 0$ as in this section, equilibrium is stable if the slope of the tangent is greater than the slope of the straight line. Therefore, the equilibrium A and C are stable, and B is unstable.

Furthermore, when x fluctuates around A, x may possibly pass from A to C over time. This result is a distinguishing feature of the binary choice model that differentiates this paper from preceding studies of multiple equilibria.

4.4 Comparative statics

This subsection analyzes how many equilibria exist when each variable takes different values.

4.4.1 Change in the nominal interest rate

We begin with considering a change in r, the nominal interest rate. Change in r for fixed λ, α, and γ is shown in Fig.6. When r is substantially low, there is an equilibrium in which most economic agents hold money. As r rises, the economy moves toward one of two equilibria—one in which many agents hold money, the other in which few agents do so. Further increase in r results in an equilibrium in which only a few agents hold money, because a large r makes holding bonds more attractive.

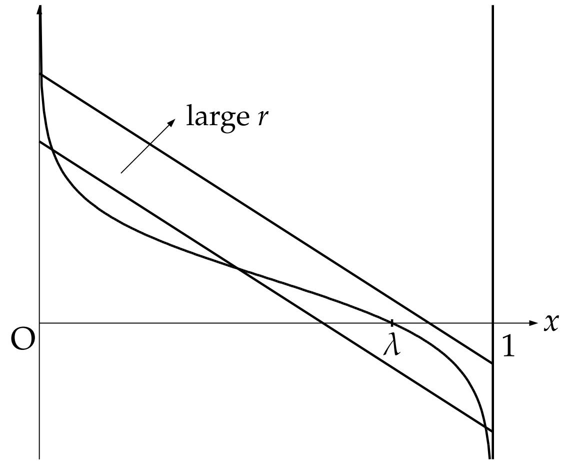

Fig.6

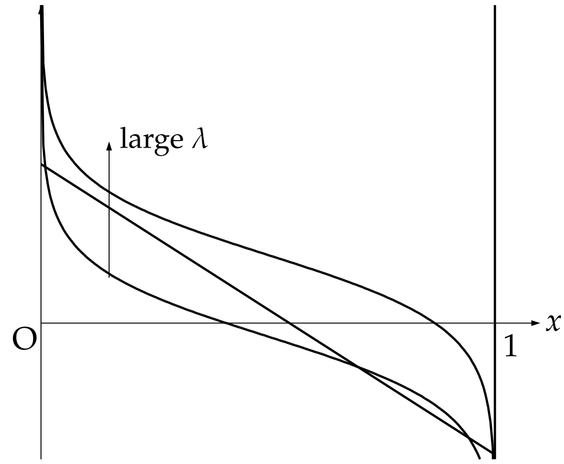

Fig.7

4.4.2 Change in the liquidity preference effect

Changes in λ, the liquidity preference effect, for fixed r, α, and γ are shown in Fig.7. When λ is fairly small, there is an equilibrium in which most economic agents hold bonds. As λ becomes large, there arise two equilibria in which many or few economic agents hold money. Further rise in the value of λ brings an equilibrium in which most agents hold money, because a higher λ indicates the superiority of holding money regardless of the value of x.

4.4.3 Change in the field effect

Fig.8 illustrates changes in α, the field effect, for fixed r, λ, and γ. When α is very large, there is an equilibrium in which most economic agents hold money. As α decreases, two stable equilibria emanate. An additional decline in α sponsors an equilibrium in which few agents hold money, because α represents the degree of the field effect on holding money.

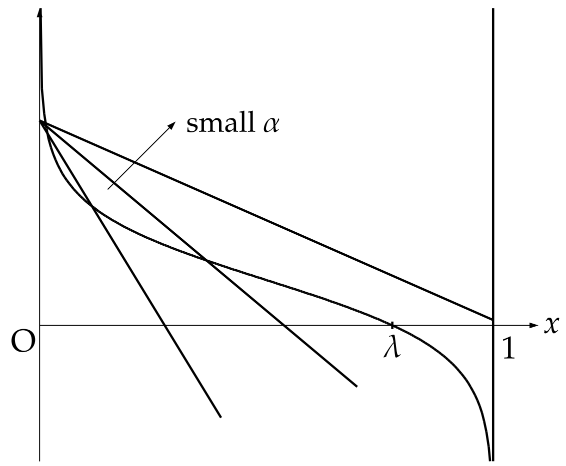

Fig.8

4.4.4 Change in the heterogeneity of expectations

Finally, consider the case in which γ, the heterogeneity of expectations, changes values for fixed r, λ, and α. As γ decreases, the straight line rotates clockwise around $(\frac{r}{\alpha}, 0)$. Fig.9(a) draws the case of $\frac{r}{\alpha} \leq \lambda$, and Fig.9(b) shows the contrary case of $\frac{r}{\alpha} > \lambda$.

When $\frac{r}{\alpha} \leq \lambda$, money holding is far superior to bond holding. In this case, most agents hold money when γ is sufficiently large. However, as the value of γ falls, the system comes to have multiple equilibria. Further decline in γ still yields multiple equilibria which locate, however, at more distant places.

We examine the latter case more thoroughly. In an extreme case in which $\gamma = \infty$, there is an equilibrium such as $x = \lambda$. Since expectations of economic agents are so dispersed that the superiority of holding either money or bond (the value of α and r) does not influence their action, and only the entropy effect works. Perversely, a very high interest rate does not affect agents' decisions.

With the decrease in γ, the system has only one solution within the range of $x < \lambda$. However, a further decrease generates three solutions of which both extremes are locally stable.[9]

[9] Let us denote $f(x) = N \ln \frac{1-x}{x} + \frac{\alpha}{\gamma} x$ and set x_0 and x_1 ($x_0 < x_1$) for two real solution of $x^2 - x + \frac{N\gamma}{\alpha} = 0$. The condition for the existence of multiple equilibria is $f(x_0) \leq \frac{r}{\gamma} - N \ln \frac{1-\lambda}{\lambda} \leq f(x_1)$.

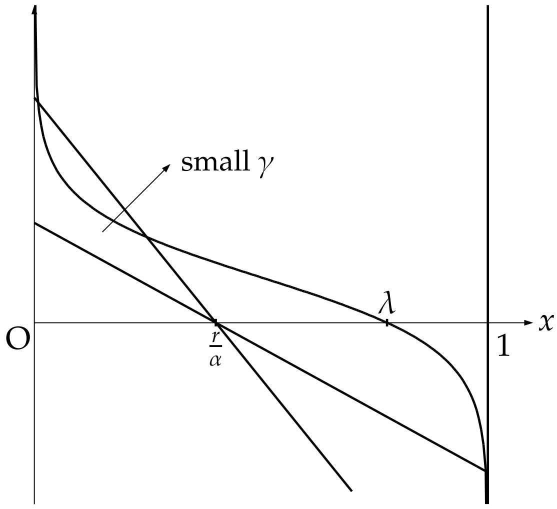

Fig.9(a)

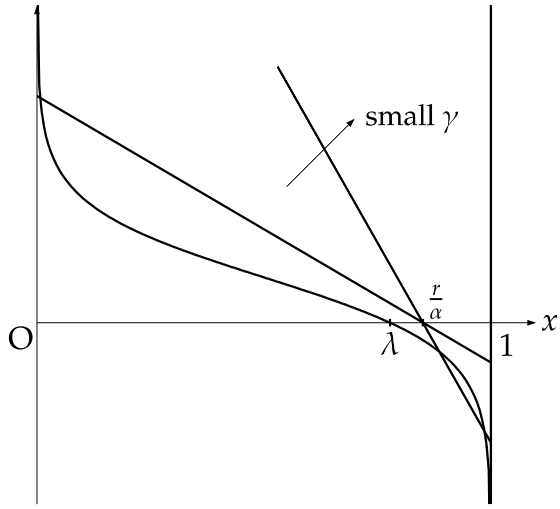

Fig.9(b)

4.4.5 Summary

To summarize, there is only one equilibrium for extreme values of r, λ, and α, but there are multiple equilibria within some ranges of the values. Each parameter represents the relative advantage of holding bonds or money. Then, most agents choose the more desirable alternative in equilibrium.

The above rule does not apply to the parameter γ. For multiple equilibria to exist, γ should be within a certain range of values. That is, the existence of multiple equilibria entails that agents' expectations are less diverse, but instead, analogous. As indicated in Section III.B, the situation in which most agents have similar expectations and want to hold money is the liquidity trap suggested by Keynes.

5 Financial crises and monetary policy

This section presents a practical interpretation of the model and discusses implications for monetary policy.

5.1 Interpretation of the model

As in Section 3, when there are no field effects on holding money, we can derive the money demand function in standard macroeconomics texts. In particular, when the heterogeneity of subjective expectations, γ, is small, or when expectations of agents are less various, the interest elasticity of money demand is quite large for relatively low nominal interest rates. This situation reflects the liquidity trap.

Keynes points out that a “variety of opinion” is fundamental for the stability of the economic system. This study has confirmed his insight in the framework of a stochastic dynamic model, showing that the liquidity trap is likely to occur when the heterogeneity of subjective expectations is small.

Section 4 extends the argument by incorporating field effects on money holding into the model, and presents the possibility of multiple equilibria. When there are three equilibria, both extremes are stable. Many economic agents hold money in one equilibrium, and few do so in the other.

When a large fraction of agents hold bonds, they are taking risks and carrying out transactions actively. In an extreme case of the above, their investment appetites are so strong that the economy has excess liquidity and reaches *the excess liquidity equilibrium*. In this equilibrium, since few people hold money, holding money is far inferior to holding bonds. Due to field effects, the merits of holding bonds are magnified by the fact that many agents hold bonds. This equilibrium may be interpreted as a bubble economy in which many investors demand illiquid assets.

In contrast, when the proportion of money holders is large, each agent avoids risk and executes transactions infrequently. The extreme case of this inactive equilibrium is *the liquidity crisis equilibrium* in which agents have weak investment appetites and prefer liquid assets over illiquid assets. Many people hoard money in this equilibrium, and the relative advantage of money over bonds is quite large because of the field effect. As a result, the majority of investors want to part with illiquid assets and hold liquid assets—money.

A large body of previous literature concerns multiple equilibria. However, most studies confine themselves to establishing their existence, leaving actual equilibration processes to exogenous forces. Through the binary choice model, this paper specifies the mechanism by which equilibrium is realized stochastically.

As stated above, the multiple equilibria in this model are interpreted as the excess liquidity equilibrium and the liquidity crisis equilibrium—namely, a liquidity crisis equilibrium lurks behind the excess liquidity equilibrium in which an economic bubble occurs. Further, the model indicates that the economy may leap from one equilibrium to the other over time. When the economy drops down to the latter equilibrium, many investors try to sell their illiquid assets and draw liquidity. Consequently, liquidity drys up. This is nothing less than a liquidity crisis.

It should be noted that multiple equilibria exist only when the heterogeneity of subjective expectations, γ, is small. Both excess the liquidity equilibrium and the liquidity crisis equilibrium may exist when agents have almost identical expectations about the future.

5.2 Field effects and financial crisis

Section 4 states that multiple equilibria may exist and that the liquidity crisis equilibrium is realized stochastically. This situation occurs when money holding becomes more attractive as the ratio of money holders increases. That is, the occurrence of multiple equilibria hinges on the existence of field effects. Therefore, if a financial crisis is interpreted as the system being trapped in the liquidity crisis equilibrium, we should clarify what condition creates field effects.

In Section 4.1, the field effect is defined as the aggregate effects from a whole population (or a fraction) of microeconomic units. What condition prompts agents' personal actions to be affected by the number of other agents who choose an option? In particular, in what situation does a decision become more attractive by the fact that many others choose it?

An example of field effects is herd behavior. Scharfstein and Stein (1990) show that an agent may mimic other agents, ignoring substantive private information. In this case, her decision is based only on the fact that many other agents are choosing that decision.

Diamond and Dybvig (1983) study the bank run equilibrium caused by strategic complementarity such that behavior of other people affects one's optimal behavior. Morris and Shin (2000) extend their argument, constructing a model in which the probability of an action taken by agents is affected by economic fundamentals. They then calculate the probability of a bank run occurring in equilibrium. When there is strategic complementarity, agents have an incentive to duplicate the actions of others. In our model, money holding becomes more beneficial as the number of agents favoring liquid money increases.

At the 2005 Federal Reserve Bank of Kansas City Economic Symposium, Hyun Song Shin analyzed past financial crises and proposed practical policies (Shin, 2005). He pointed out that liquidity had attributes of a public good that originated from the diversity of intended actions. Referring to the accident when the Millennium Bridge in London was shaken violently by the synchronized movement of pedestrians, he insisted that a financial crisis occurred when market participants acted uniformly. He cited the 1987 stock market crash as an example of such crises, and stated that dynamic hedging let market participants act uniformly, precipitating the market crash. An agent who adopts dynamic hedging has an incentive to sell more stocks when she observes that others are selling and stock prices are falling. A chain reaction of selling is an example of field effects described in our model. Shin also claimed that not only dynamic hedging but also price-sensitive incentive schemes, price-sensitive risk-management systems, and mark-to-market accounting might induce investors to take synchronized actions that amplified a crisis.

Allen and Carletti (2008) theoretically analyze the effect of mark-to-market accounting on asset prices. They show that when the market lacks sufficient liquidity, asset prices may reflect the amount of liquidity available rather than the asset's future earning power. If the mark-to-market rule is enforced in that situation, banks, incurring accounting loss in their long-term assets, are forced to sell them at distressed prices. Further, Brunnermeier and Pedersen (2009) explain with a theoretical model that margins in financial transactions may destabilize the system, leading to liquidity spirals.

These studies show that there are various systems in which illiquid assets become more attractive when others hold illiquid assets, and liquid assets become more attractive when others hold liquid assets. Such systems would strengthen the field effects analyzed in this paper.

5.3 Implication for monetary policies

Both Japan and U.S. have experienced an abrupt liquidity crisis following the bubble boom—Japan through its "lost decade" and U.S. through the financial crisis triggered by the subprime crash. Our model suggests that the economy possibly had multiple equilibria—one in which few agents held money and the other in which many agents held money—and transited from the former to the latter. If the financial crisis can be interpreted as a plummet to the latter equilibrium, what economic policies would be needed to escape the equilibrium? This section proposes the remedial measures which manipulate parameters in our model. This study refers them r-policy, λ-policy, α-policy, and γ-policy.

5.3.1 r-policy

In the liquidity crisis equilibrium, the economy is in the state of depression. Agents are risk averse, and investment is dormant. To counter the situation, monetary authorities usually lower the nominal interest rate, r. However, Fig.6 illustrates that lowering the nominal interest rate shifts the straight line downward, moving the liquidity crisis equilibrium rightward. As a result, the ratio of money holders rises further. Thus, lowering the interest rate inadvertently leads people to be locked into money holding. When field effects are present in financial markets, the conventional and commonsense monetary policy may be ineffective.

5.3.2 λ-policy

Next, let us consider the policy that manipulates λ, the liquidity preference effect. As shown in Fig.7, if λ is sufficiently small, the liquidity crisis equilibrium disappears, and the economy has only the equilibrium shown in the left of the figure.

What policy can diminish λ? Unlike α, λ indicates the benefits of money holding independently of the number of money holders. Therefore, a policy that reduces the attractiveness of holding money relative to bonds should be implemented.

Bond holding may be unattractive partly due to what might be called Knightian uncertainty in the economy. That is, people may avoid risky assets and escape to money holding when economy's future prospect are vague. Keynes (1937) notes:

> Because, partly on reasonable and partly on instinctive grounds, our desire to hold Money as a store of wealth is a barometer of the degree of our distrust of our own calculations and conventions concerning the future. Even tho this feeling about Money is itself conventional or instinctive, it operates, so to speak, at a deeper level of our motivation. It takes charge at the moments when the higher, more precarious conventions have weakened. The possession of actual money lulls our disquietude; and the premium which we require to make us part with money is the measure of the degree of our disquietude.

Thus, if people demand money because they view the future as precarious, measures that alleviate uncertainty, albeit indirectly, may heal an economy in the liquidity crisis equilibrium.

Meanwhile, the Gesell carry tax on currency taken up in Keynes (1936) may be another and more direct effective tool. It increases the cost of money holding, reduces the attractiveness of money, and may encourage people to hold bonds. It may be an effective tool to help the economy to get out of the liquidity crisis equilibrium.

5.3.3 α-policy

We next consider the controllability of α, the field effect. If α is sufficiently small, there may be only the left-side equilibrium. How can policymakers decrease field effects? As described in Section 5.2, there are various economic systems which would reinforce field effects. These systems should be amended.

The first possibility is a regulatory change that alters investment managers' incentives. This idea was proposed by Raghuram G. Rajan at the Federal Reserve symposium mentioned above (Rajan, 2005). He argued that fund managers who were evaluated against a common benchmark like the S&P 500 Index might engage in herd behavior, knowing their poor performance would be excused when their benchmark also performed poorly and that they would be dismissed if they underperformed as a result of bucking the trend. Presupposing these behaviors, Rajan (2005) asserted that the incentive mechanism should be reconstructed to prevent investment managers from taking excessive risks with myopic strategies.

The second potential way to diminish field effects may be to reform the accounting system. Again at the 2005 Fed symposium, Hyun Song Shin stressed that mark-to-market accounting should be abolished (Shin, 2005). Facing the subprime loan crisis, Brunnermeier et al. (2009) suggested more concrete policies: they proposed a mark-to-funding instead of mark-to-market requirement, presuming that externalities arise from the bank's balance sheet effect. Their proposal intends to reduce the externalities and prevent a "loss spiral" by modifying accounting rules in a manner that encourages financial institutions to hold long-term bonds.

In considering any financial system, we should remember there may be field effects and that market participants may synchronize behavior, destabilizing the whole economic system even if their actions are rational. Financial regulations should serve not only the security of the individual agent but also the stability of macroeconomic system.

So far, we have examined system modifications that abate field effects. Alternatively, monetary policies may also be effective. As Shin (2008) mentions, if shocks may be amplified in a financial system with interlinked claims and obligations, the preferred policy is to sever the links. In this vein, central banks as *the lender of last resort* may play the decisive role in preventing a liquidity shortage from spreading to other agents.

5.3.4 γ-policy

Finally, let us consider the policy related to the heterogeneity of subjective expectations, γ. Recall that it is the uniformity among agents' expectation that ultimately causes financial crises. If most agents have pessimistic forecasts about the future, traditional monetary policy of lowering the interest rate may have little effect. Called for is a policy that directly affects expectations and engenders more divergent views about the future.

The Japanese economy stagnated during its "lost decade" from the 1990s to the early 2000s.[10] Although the Bank of Japan has persisted in its *zero interest rate policy*

[10] Aoki, Yoshikawa, and Shimizu (2005) analyze theoretically by using binary choice model and explain long stagnation by rising the degree of uncertainty.

and *quantitative monetary easing policy* since the end of 1990s, some of academics and policymakers remain skeptical about the validity of these policies. We now argue, however, that these policies may be useful if they can induce people to expect that relaxing policies will be continued for a long duration. Oda and Ueda (2007) insist these policies actually worked because the Bank of Japan's commitment to the low interest rate became more credible as time passed and began to affect expectations. Indeed, our model suggests such policies can be effective if they mitigate the uniformity of pessimistic expectations among investors.

In the subprime financial crisis of 2007, a liquidity drain suddenly erupted. This study interprets this phenomenon as being caused by the synchronization of expectations. A large majority of people plunged uniformly into pessimistic views of the future, holding more money and thereby driving the economic system into the liquidity crisis equilibrium. This interpretation implies that the so-called quantity easing by the central bank was an appropriate policy. For an economy to escape a liquidity crisis equilibrium, an extraordinary quantity of liquidity is needed. In addition, monetary easing may directly influence people's expectations if the central bank proclaims it will definitely protect the financial system.

History shows on occasions that expectations of individuals within a system produce harmful effects on the whole system. As Diamond and Dybvig (1983) and many subsequent studies maintain, uniformly pessimistic expectations may create a bank run equilibrium even if banks have no solvency problem. Once the bank run equilibrium is in place, sound banks may have solvency problems and be involved in a panic. Thus, pessimistic expectations can produce solvency problems and may eventually damage the entire banking system. In this case, the central bank's commitment to protecting the whole financial system may abate pessimism and forestall its spread, preventing the system from falling into the bank run equilibrium. Recognizing the analytical results of past dire experiences, policymakers must remember that the economic system is vulnerable to the latent power of people's expectations.

6 Concluding Remarks

This paper considers people's choice between money and bonds using the binary choice model. The model indicates a liquidity trap might exist when economic agents share uniform expectations about the future. In a sense, this study constructs a theoretical model of the argument by Keynes. Further, the model demonstrates the possibility of multiple equilibria when field effects exist. Few agents hold money in the excess liquidity equilibrium and most do so in the liquidity crisis equilibrium. The former is a bubble economy, and the latter is an economy in a liquidity trap.

In the model, equilibrium is realized not exogenously but stochastically. This distinguishing feature of the model differentiates our study from earlier studies of multiple equilibria. It allows us to show that the economy may stochastically fall into the liquidity trap from a bubble, and it clarifies the mechanism of financial crises. In addition, this study presents implications for monetary policy. First, lowering the interest rate may inadvertently encourage more people to hold money, locking the system in the liquidity crisis equilibrium. Second, the effective policy is one that reduces uncertainty and alters people's expectations. Finally, the study analyzes the significance of monetary policy in the U.S. during the subprime crisis and in Japan during its lost decade.

Economic studies have not thoroughly analyzed the impact of the heterogeneity among economic agents or the heterogeneity of expectations on the macroeconomic system. However, after the recent financial crisis, it is broadly recognized that uniform behavior among agents may stagger the whole system.

This study is the first step toward considering and incorporating the heterogeneity of expectations into the model. While the apparatus of this study much owes to the entropy effect and field effects introduced by stochastic dynamic modeling, few studies have attended to these effects. Extending the argument present here, constructing more empirically robust models, and providing suggestions for practical and implementable policies remain for future studies.

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